Math 1C Midterm 3

Fri Jun 17, 2022 DUE Mon Jun 20, 2022 @ 2pm in Canvas

SCORE: _____ / 175 POINTS

INSTRUCTIONS

[A] For this test, you may consult your lecture notes for sections 10.1-10.4, 12.1-12.5 the Zoom recordings for sections 10.1-10.4, 12.1-12.5 linked on the instructor's website, your textbook sections 10.1-10.4, 12.1-12.5 and the homework you did for those sections. You may **not** simply use the answer of a textbook exercise as justification – you must write a complete solution for that answer in your work.

You may **not** use any other material located on the instructor's website nor covered in any other sections of your textbook.

You may **not** consult any person, nor any written/printed material, website, software, app or other electronic resource, nor any calculator (unless instructed), computer, phone or other electronic device.

[B] For each derivative that requires the product, quotient and/or chain rule, you must show all the factors and terms from the rule before you simplify.

eg. $\frac{d}{dx}\frac{xe^{2x}}{1+e^x} = \frac{(1(e^{2x})+xe^{2x}(2))(1+e^x)-xe^{2x}(e^x)}{(1+e^x)^2}$ ⬅ **THIS STEP IS REQUIRED**

$= \frac{e^{2x}((1+2x)(1+e^x)-xe^x)}{(1+e^x)^2} = \frac{e^{2x}(1+2x+e^x+xe^x)}{(1+e^x)^2}$

For each integral that requires $u-$substitution, you must show the value of u and the resulting integral in terms of u.

eg. $\int 3x(2x^2-1)^5\,dx$ $= \int \frac{3}{4}u^5\,du \quad (u=2x^2-1)$ $= \frac{1}{8}(2x^2-1)^6+C$ ⬅ **MINIMUM WORK REQUIRED**

For each integral that requires integration by parts, you must show the work (table method OK).

For each integral that requires polynomial long division, partial fractions decomposition or trigonometric substitution, you must show all work.

The general rule is that, **if you can't do the work in your head without writing something down, write it in your test**.

[C] **Handwrite** your solutions to the questions on clean 8½″ × 11″ paper (or equivalent).

[D] Your solutions to the questions must be in the same order as the questions in this test. (You may write the solutions to each question on separate pages, and sort them in order afterwards.)

[E] You do not need to copy the questions onto your paper. Just show your organized and clearly written work and final answers.

[F] Writing which is illegible to the instructor will earn 0 points.

[G] All final algebraic answers must be completely simplified to receive full credit.

[H] All work must be properly algebraically justified and use proper mathematical notation as shown in lecture, not simply based on scratch work, intuition or "handwaving".

Imagine yourself tutoring a struggling Math 1C student. They should be able to understand your algebraic reasoning based on your writing alone.

[I] Upload a **single** clear & legible PDF of your completed test to Canvas no later than Mon Jun 20 @ 2pm Pacific Time.

The solution will be posted to my website shortly after that time, and all work submitted after that will earn 0 points.

QUESTIONS

[1] Legibly **handwrite** the text from the box below (do **not** write in cursive), and sign your name directly below your writing.
If you skip this step or your writing is illegible to me, your quiz will not be counted for credit.

> "I am a principled and honorable person who can be trusted.
> I pledge to uphold the De Anza College Student Code of Conduct.
>
> By signing below, I confirm that the work shown on this test is strictly my own.
> Other than the resources listed in Instruction [A] of this test,
> I did not consult any person, nor any written/printed material, website, software, app or other electronic resource,
> nor any computer, phone, calculator or other electronic device."

NOTE: The De Anza College Student Code of Conduct can be found at
https://go.boarddocs.com/ca/fhda/Board.nsf/goto?open&id=9U2UC77B2DA5

0 points for all questions in which it is evident you used a calculator for any part of your work (unless explicitly allowed)
Use fractions, radicals, e, π, logarithms, trigonometric functions and their inverses, **not decimals**

Read Instruction [A] above for which resources are **not** to be used during your test and how certain resources are to be used

Read Instructions [B] and [H] above for required standard of writing

Run all mandatory sanity checks discussed in lecture

[2] **[21 POINTS]**

Convert the polar equation $r = \dfrac{\csc\theta}{\cos 2\theta}$ to rectangular
using the process in the Polar Equation Conversion handout on the instructor's website.

[3] **[10½ POINTS]**

Convert the rectangular coordinates $(-35, -21)$ to polar.
Your final answer may involve an inverse trigonometric function.
REMINDER: Do not use a calculator or any other form of technology.

[4] **[49 POINTS]**

Consider the points $P(3, -5, -3)$, $Q(-1, 1, -2)$ and $R(-1, 2, -3)$.
Let $\vec{u}$ be the vector with terminal point R and initial point Q.
Let $\vec{v} = \overrightarrow{PQ}$.

[a] Find the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$.

[b] Is the angle between $\vec{u}$ and $\vec{v}$ acute, obtuse, right, or none of the above ?
REMINDER: Do not use a calculator or any other form of technology.

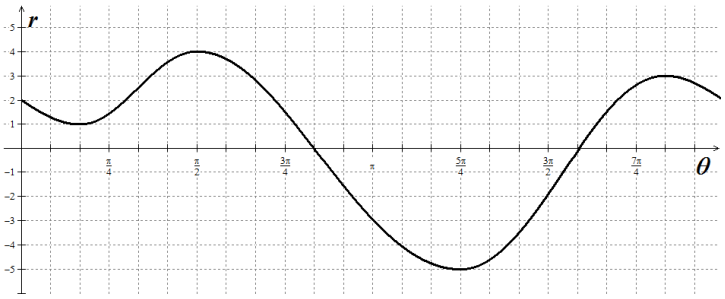
[c] Find two vectors of magnitude 12 which are perpendicular to both $\vec{u}$ and $\vec{v}$.

[d] Write $\vec{v}$ as the sum of a vector perpendicular to $\vec{u}$ and a vector parallel to $\vec{u}$.

[5] [21 POINTS]

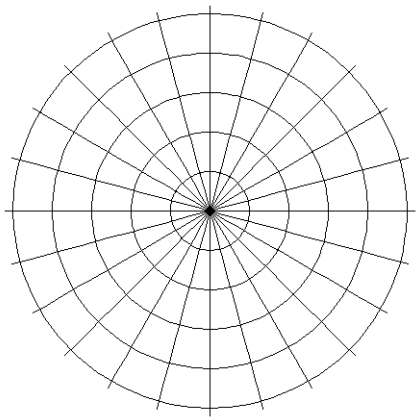
EJ and FJ are working on a polar curve graphing exercise using the Polar Graphing 2 handout on the instructor's website.

They correctly sketch the graph of $r = f(\theta)$ on the Cartesian plane, shown below.



NOTE: On the graph above, the distance between each pair of adjacent vertical gridlines is $\frac{\pi}{12}$.

Complete the remainder of the process shown in the handout and sketch the polar graph of $r = f(\theta)$ on the polar graph paper below, using as much of the polar grid as possible. Your graph should be **smooth** (no "corners").



NOTE: **0** points for a graph without the analysis shown in the handout.
You must use the polar graph paper above.

[6] [14 POINTS]

Test the polar graph $r = 2\cos\theta - \cos 2\theta$ for symmetry about $\theta = \frac{\pi}{2}$
by running the symmetry tests discussed in lecture (included in the Polar Graphing handout on the instructor's website).
State clearly what conclusions result from each test, as well as the final conclusion.
Run as few tests as necessary to justify that final conclusion.
REMINDER: Do not use a calculator or any other form of technology.

[7] [24½ POINTS]

Let $\vec{u}$ and $\vec{v}$ be vectors which are perpendicular to each other, where the magnitude of $\vec{u}$ is 5, and the magnitude of $\vec{v}$ is 7.

NOTE: In the questions below, "vector algebra" refers to the properties in the red boxes in sections 12.2, 12.3 and 12.4 which do **not** involve the component form of vectors.
eg. the red box at the top of page 840 **is not** vector algebra, whereas the red box at the bottom of page 840 **is** vector algebra.

[a] Find $(2\vec{u} - 3\vec{v}) \cdot (\vec{v} - 4\vec{u})$ using vector algebra.

[b] Completely simplify $(\vec{v} \times \vec{u}) \times \vec{u}$ using vector algebra.

[c] Find $\left\|(\vec{v} \times \vec{u}) \times \vec{u}\right\|$.

[8] **[21 POINTS]**

Find $\frac{d^2y}{dx^2}$ for the polar curve $r = 2\cos\theta$. Use identities to simplify at every stage to significantly reduce the amount of work.

HINT: How was the formula for $\frac{dy}{dx}$ for polar curves justified ?

Do NOT convert the polar equation to rectangular. Your final answer must be a function of θ only.

[9] **[14 POINTS]**

Let P be the point with polar coordinates $(-8, \frac{19\pi}{13})$.

[a] Find a pair of polar coordinates for P such that $r > 0$, $\theta \geq 0$ and $|\theta|$ is as small as possible.

[b] Find a pair of polar coordinates for P such that $r < 0$, $\theta \leq -2\pi$ and $|\theta|$ is as small as possible.

[c] Find a pair of polar coordinates for P such that $r > 0$, $\theta \leq -2\pi$ and $|\theta|$ is as small as possible.

[d] Find a pair of polar coordinates for P such that $r > 0$, $\theta \geq 2\pi$ and $|\theta|$ is as small as possible.